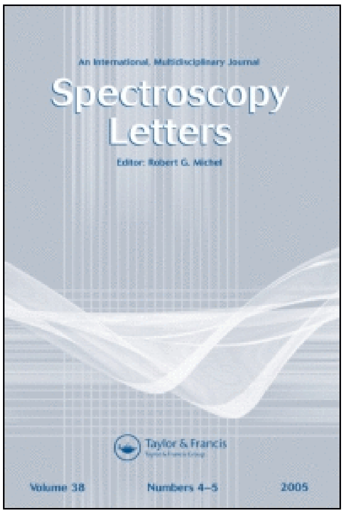

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Chemistry in Lasers. XI. Non-Classical Aspects of Fusion: The Zhabotinsky Effect and Communications Theory

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CHEMISTRY IN LASERS. XI.

NON-CLASSICAL ASPECTS OF FUSION:

THE ZHABOTINSKY EFFECT AND COMMUNICATIONS THEORY.

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ABSTRACT

Developed from the application of operational mathematics and Markoff-theory in quantum mechanics [CA:81:8225v], transform space treatment of Einstein diffusion [CA:82:49699s], and acousto-opic effects in lasers [CA:82:162686y], the paper examines non-classical aspects of laser driven fusion presenting favorable options which remain generally inexplored.

(End of Abstract)

The problem of providing sufficient energy densities to allow economically successful laser driven fusion is a source of continuing innovation in the scientific community. Basic and applied research on lasers has maintained, or if possible even accelerated, the kaleidoscopic effect of new ideas[1-5]. Keeping in tune with current developments could be considered an almost full-time task, and a somewhat radical departure from on-going activity a hard to justify expenditure of time and resources. Yet the scientific approach must remain cognizant of the fact that progress is often 'around the corner' rather than 'straight down the road'; the three examples that one may recall pertain to electricity, aviation, and lasers. In the early days electricity was regarded as a mere scientific curiosity, a non-sequitur in the realm of competition with the steam engine and gas light. Prior to the successful flight of the Wright brothers, a learned academician has published a lengthy communique in which he demonstrated that mechanical flight of heavier than air devices was scientifically impossible. Finally, the best He-Ne laser line is one that was previously regarded quite unfeasible, whereas the prime candidate lines turned out to be unfeasible. In essence, then, we should clearly distinguish between re-assessing our options, and re-ordering our priorities. We should continually attach utmost significance to pursue the former, so that we can undertake the latter most efficiently whenever such action is mandated. A typical set of options related to energy concern nuclear fission, laser driven fusion, and solar power. Volatile as the issues may be, the scientific framework calls for continuous updating and reassessing our options. The ensuing material relates to some special aspects of the interaction between matter and radiant energy.

In pursuing the problem of fusion via laser driven implosion[6], a very complex situation presents itself. Within a time span that is extraordinarily brief to begin with, one should cope with changing from solid to liquid to plasma. Temperatures reaching about 10^6 ^oK, density $\sim 10^{26}$ particles /cm^3, inertial confinement time approximately 10^{-10} - 10^{-12} seconds are but part of the physical constraints; absorption of laser light in the critical density region requires about one millimeter plasma depth for ~ 1 μmeter irradiation, and 100 mm plasma depth for 10μ laser light. By characterizing the system in several ways, we can proceed to cite some new options in extending plasma confinement time -- the variable that should approach the fusion reaction time, which is less than a microsecond. (A great discrepancy between plasma confinement time and fusion reaction time, of course, reduces the effective sample size for energy release.)

The application of m a p p i n g in the characterization of quantum states as a f i n i t e c o u n t a b l e M a r k o v c h a i n has been cited in the Abstract, and we shall first expand the related mathematical framework.

Topologically, the boundary between the imploding and exploding part of the sample should function to provide two subsets of the topological space X that are separated or disconnected. If we let the two subsets be A and B, separation is achieved iff

$$A \cap \bar{B} = \emptyset \qquad \text{and} \qquad \bar{A} \cap B = \emptyset \qquad \{1\}$$

whereas subset A can be considered disconnected if there exist open subsets D and E of X such that $A \cap D$ and $A \cap E$ are disjoint non-empty sets whose union is A, $D \cup E$ being the disconnection of A. Formally, $D \cup E$ is a disconnection of A iff

$$\begin{aligned} &A \cap D \neq \emptyset \\ &A \cap E \neq \emptyset \\ &A \subset D \cup E \\ &D \cap E \subset A^c . \end{aligned} \qquad \{2\}$$

Next we may consider compression and implosion in terms of p a t h requirements. The homotopy H will, for example, compress the domains F $(F : I^2 \to X)$ and G $(G : I^2 \to X)$ if $H : I^2 \to X$ follows

$$H(s,t) = \begin{cases} F(s,2t) & \text{if } 0 \leq t \leq \frac{1}{2} \\ G(s,2t-1) & \text{if } \frac{1}{2} \leq t \leq 1 . \end{cases} \qquad \{3\}$$

To represent compression sequentially, c o n t r a c t i n g m a p p i n g can be employed. If X is a metric space, a function $f : X \to X$ is a contracting mapping if there exists a real number α, $0 \leq \alpha \leq 1$, such that for every $p,q \in X$,

$$d(f(p),f(q)) \leq \alpha d(p,q) < d(p,q) , \qquad \{4\}$$

i.e. the distance between the images of any two points is less than the distance between the points.

In relation to contracting mapping it is important to establish that a complete metric space converges to a single point $p \in X$, even though c o m p l e t e n e s s is not a topological property.

To demonstrate the convergence of a complete metric space to a single point, we let a_0 be any point in X. By setting

$$a_1 = f(a_0), \quad a_2 = f(a_1) = f^2(a_0), \ \dots, \ a_n = f(a_{n-1}) = f^n(a_0), \ \dots, \qquad \{5\}$$

our claim is that $\langle a_1, a_2, a_3, \dots \rangle$ is a Cauchy sequence. Note that

$$d(f^{s+t}(a_0), f^t(a_0)) \leq \alpha d(f^{s+t-1}(a_0), f^{t-1}(a_0)) \leq \dots \leq \alpha^t d(f^s(a_0), a_0) \leq \alpha^t [d(a_0, f(a_0)) + d(f(a_0), f^2(a_0)) + \dots + d(f^{s-1}(a_0), f^s(a_0))]. \qquad \{6\}$$

But

$$d(f^{i+1}(a_0), f^i(a_0)) \leq \alpha^i d(f(a_0), a_0), \quad \textbf{hence}$$

$$d(f^{s+t}(a_0), f^t(a_0) \leq \alpha^t d(f(a_0), a_0)(1 + \alpha + \alpha^2 + \dots + \alpha^{s-1}) \leq \alpha^t d(f(a_0), a_0)[1/(1-\alpha)] \qquad \{7\}$$

since $$(1 + \alpha + \alpha^2 + \dots + \alpha^{s-1}) \leq 1/(1-\alpha). \qquad \{8\}$$

If we let $\epsilon > 0$ and set

$$\delta = \begin{cases} \epsilon(1-\alpha) & \text{if } d(f(a_0), a_0) = 0 \\ \epsilon(1-\alpha)/d(f(a_0), a_0) & \text{if } d(f(a_0), a_0) \neq 0, \end{cases} \qquad \{9\}$$

because $\alpha < 1$, $\exists\, n_0 \in N$ such that $\alpha^{n_0} < \delta$; now if $r \geq s > n_0$,

$$d(a_s, a_r) \leq \alpha^s [1/(1-\alpha)] d(f(a_0), a_0) < \delta [1/(1-\alpha)] d(f(a_0), a_0 \leq \epsilon \qquad \{10\}$$

and $\langle a_n \rangle$ is a Cauchy sequence. Because X is complete, $\langle a_n \rangle$ converges to a point, say, $p \in X$. We claim that $f(p) = p$; since X is sequentially continuous,

$$f(p) = f(\lim_{n\to\infty} a_n) = \lim_{n\to\infty} f(a_n) = \lim_{n\to\infty} a_{n+1} = p. \qquad \{11\}$$

Finally we show that p is unique. Suppose $f(p) = p$ and $f(q) = q$; then $d(p,q) = d(f(p),f(q)) \leq \alpha d(p,q)$. But $d(p,q) = 0$ since $\alpha < 1$, hence $p = q$.

Some of the previous work cited in the A b s t r a c t was also related to convergence; the treatment was rather one-sided in the present context, assuming ideal cylindrical diffusion and the establishment of monotonic density gradients towards the center of laser interaction, mathematically prescribed in terms of B e s s e l f u n c t i o n s. The reason the previous treatment is considered inadequate at the present time is that it was based on boundary conditions similar to those applying to ordinary heat flow from an isotropic reservoir, which is an unlikely approximation for laser driven implosion and fusion. On the other hand, the current problem presents synchronous needs for at least two of the fields of dynamics: c l a s s i c a l and q u a n t u m. In Part IV of this series it was pointed out that macroscopic representations such as taking recourse to an oscillator feedback loop analogy have been remarkably successful in predicting and rationalizing the operation of tuned dye lasers for example, whereas statistical considerations on the molecular level[7] require a more serious departure from the usual treatment to account for the same physical observations. Perhaps not surprisingly, there is continuing development in the descriptive disciplines themselves[8,9], quite apart from the complex applications. It is interesting to note, not merely as scientific history but also in order to develop a cohesive view of our subject, that less than a hundred years ago the continuum representation was more freely used to treat chemically related problems and systems[10-12], and it is apparent that the disproportionate share of our attention claimed at the present time by the microscopic approach is a dubious blessing if the system has important continuum properties.

Only in recent years has chemistry come to investigate[13] in detail such momentum related phenomena as the Zhabotinsky effect, sheer gradient effects on it, Liesegang rings and periodic precipitation phenomena, and the amplification of sound in chemical reactions. Thus we have an impetus for further exploring the total m o m e n t o f m o m e n t u m,

$$N_i(t) = \int_V \epsilon_{ijk} X_j \rho v_k dV \qquad \{12\}$$

or

$$\mathbf{N} = \int_V (\mathbf{X} \times \rho\mathbf{V}) dV , \qquad \{13\}$$

for the introduction of oscillatory effects into optically driven implosion and fusion presents an option for very significant extension of effective confinement time. Proper description can be formulated from the spatial gradient of the instantaneous velocity field that defines the v e l o c i t y g r a d i e n t t e n s o r, Y_{ij} (or $\mathbf{Y}$) as

$$Y_{ij} = \tfrac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \tfrac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) \qquad \{14\}$$

or

$$\mathbf{Y} = \tfrac{1}{2}(\mathbf{V}\nabla_x + \nabla_x\mathbf{V}) + \tfrac{1}{2}(\mathbf{V}\nabla_x - \nabla_x\mathbf{V}) \qquad \{15\}$$

The second term in Equations 14 and 15 is of particular interest to us; the term represnts the skew symmetric v o r t i c i t y- or s p i n t e n s o r, which is the material derivative of the E u l e r i a n l i n e a r r o t a t i o n t e n s o r, i.e.

$$\frac{d\Omega}{dt} = \frac{1}{2}(\mathbf{V}\nabla_x - \nabla_x\mathbf{V}) \qquad \{16\}$$

A large value of the v o r t i c i t y v e c t o r, $\boldsymbol{\gamma}$, which is a c u r l of the velocity field, related to the vorticity tensor as

$$\boldsymbol{\gamma} = \nabla_x \times \mathbf{V}, \qquad \{17\}$$

will allow target sample deformation with extended confinement time even in the absence of the above mentioned periodic effects (of which there is a paucity of available observation, and are also less well understood).

Acoustic effects were cited in the A b s t r a c t as being relevant to some types of lasers, and sonic effects may also play an important role in plasma behavior. The smallest wavelength of sound is set by thermal lattice vibrations beyond which the material can not follow the input sound, hence is about 2 Å only! Although formally similar treatment of acoustical, electrical, and mechanical problems has been in the literature for a number of years (consider e.g. the H e l m h o l t z r e s o n a t o r in the acoustical, and electrical and mechanical analogue representations), our state of predictive ability for very dense and unstable plasmas is quite inadequate. For chemists unschooled in acoustics it may be worthwhile to gain a better feeling for the nature of acoustic propagation by delineating that the notion of spherical waves propagating from a common origin is of limited significance. The d i r e c t i v i t y f u n c t i o n [sin(ka$\underline{\sin}$ β)/ka$\underline{\sin}$ β] expresses non-uniform radiation characteristics of the source, giving rise to m a j o r and m i n o r l o b e s for example. The lack of information on self-focussing in the present case is very regrettable--the acoustic perturbations may not have a markedly deleterious effect on plasma stability in such systems, and in turn with the prospect of extending the confinement time by orders of magnitude, new target samples holding a promise for strong self-focussing could be systematically tested. There is very little room for doubt that communications theory has a vital and only partly explored role in describing system properties and response, aimed at extending plasma confinement by vortex or periodic effects. R o o t l o c u s analysis of the time domain response may uncover centers for oscillatory effects, or for monotonic phase angle dependent constriction as given by the N y q u i s t p l o t of $e^{-Tj\omega} \cdot [j\omega + p]^{-1}$.

In a physical situation where propagation of electromagnetic energy, sonic energy, particles, and bulk (sample) plasma are all co-temporal and co-spatial within the small target volume, descriptions can become multifarious and uncertain in the textbook sense; it is from the experimentalist viewpoint that the multifariousness brings a positive note: the expansion of sample confinement time in laser driven fusion should be actively sought, utilizing some unaccelerated, the kaleidoscopic effect of new ideas[1-5] or marginally explored effects. In the days of large institutes devoting large summs and major effort to related problems, it perhaps seems to carry a touch of audacity to propose progress along the lines of little-explored effects; such has not been the intent in presenting these remarks.

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